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with
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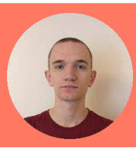

RUNAWAY SPINS IN BINARY BLACK HOLES

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INTRO

- **Black holes** (BHs) are warps in spacetime so strong that they consume ALL things that get too close.
- In BH **binary** systems two BHs orbit each other.
- The emission of **gravitational waves** (GWs) leads the BHs to inspiral and eventually merge.
- The GW signal from a merging binary BH was first detected by LIGO in 2015:

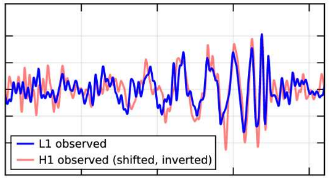


FIG 1: *The signal GW150914, the first ever observation of GWs [2].*

PRECESSION

- Consider a spinning top. Unless spun perfectly upright, it wobbles in a cone as it rotates – this is called **spin precession**.
- BHs precess if the spins are not orthogonal to the orbit, significantly altering any GW signal.
- There are four of these aligned spin configurations:

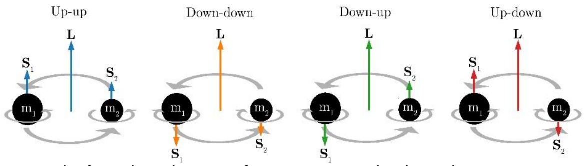


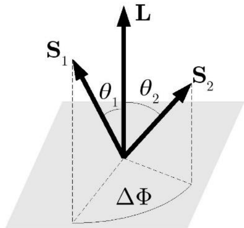
FIG 2: *The four aligned spin configurations.* L = *orbital angular momentum,* S = *spin and* m = *mass.*

- Are these systems **stable**? Or does spin precession change the configuration as the BHs evolve?

THE MATHS

1 DEFINITIONS

- Mass ratio $q = m_2/m_1$, total mass $M = m_1 + m_2$, Kerr parameters $\chi_1 = m_1^2|\boldsymbol{S}_1|$, $\chi_2 = m_2^2|\boldsymbol{S}_2|$.
- The spin vectors and their relative orientations are defined in the following figure and the evolution of binary BH spins is governed by the following equations:



$$\frac{d\boldsymbol{S}_1}{dt} = \boldsymbol{\omega}_1 \times \boldsymbol{S}_1$$

$$\frac{d\boldsymbol{S}_2}{dt} = \boldsymbol{\omega}_2 \times \boldsymbol{S}_2$$

$$\frac{d\boldsymbol{L}}{dt} = \boldsymbol{\omega}_L \times \boldsymbol{L} + \frac{dL}{dt}\hat{\boldsymbol{L}}$$

FIG 3: *The relative orientations of the spin vectors* $\boldsymbol{S}_1$ *and* $\boldsymbol{S}_2$ *with the orbital angular momentum* $\boldsymbol{L}$*, and the angles* θ_1 *and* θ_2 *between them.*

2 PERTURBATIONS

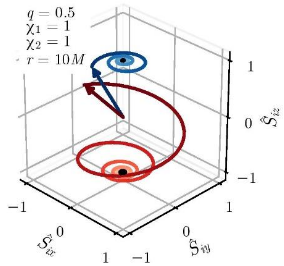
- To test the stability of the aligned spin configurations, we introduce small perturbations to the spin directions.
- This means the two spin vectors are slightly misaligned with the orbital angular momentum. We measure this misalignment with a parameter ε.
- The perturbation evolves to leading order as a **simple harmonic oscillator**:

$$\frac{d^2\varepsilon}{dt^2} + \omega^2\varepsilon \simeq 0$$

- The evolution of the perturbation depends on ω^2:
  - $\omega^2 > 0 \implies$ the spins are **stable**
  - $\omega^2 < 0 \implies$ the spins are **unstable** and begin to precess
  - $\omega^2 = 0 \implies$ transition from **stability to instability**

RESULTS

- From ω^2, we find that only the **up-down** configuration (see **FIG 2**) is unstable to precession.
- Those binaries become unstable when the distance r between the BHs shrinks to values between:



$$r_+ > r > r_-$$

$$r_\pm = \frac{\left(\sqrt{\chi_1} \pm \sqrt{q\chi_2}\right)^4}{(1-q)^2}$$

FIG 4: *The evolution of the two BH spin vectors after an up-down binary encounters the instability.*

- The **endpoint** of the unstable evolution is given by:

$$\cos\theta_1 = \cos\theta_2 = \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}$$

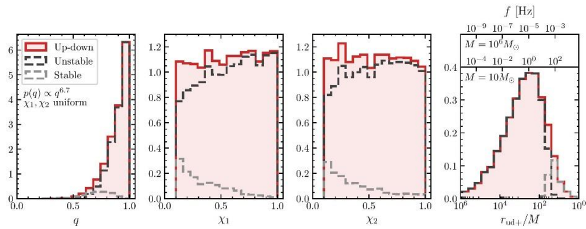


FIG 4: *Astrophysical population of up-down BH binaries. Unstable binaries make up most of the population (left three panels) and undergo the instability before reaching the LIGO sensitivity window (frequency* $\gtrsim 50$ Hz*, right panel).*

REFERENCES

[1] **M. Mould** and D. Gerosa, arXiv:2003.02281 [gr-qc] (2020).

[2] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 061102 (2016).

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